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Coefficient Estimates for Certain Subclass of Bi-Univalent Functions Obtained With Polylogarithms

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Abstract
In the present work, the author determine coefficient bounds for functions in certain subclasses of analytic and bi-univalent functions. Several corollaries and consequences of the main results are also considered. The results, which are presented in this paper, generalize the recent work of Srivastava et al. [21].

Keywords: *Analytic function; Bi-univalent function; coefficient bounds; polylogarithm function;*

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1. INTRODUCTION

Let
$$\mathbb{R} = (-\infty, \infty)$$
be the set of real numbers, $\mathbb{C}$ be the set of complex numbers and
$$\mathbb{N} = \{1, 2, 3, \cdots\} = \mathbb{N}_0 \cup \{0\}$$
be the set of positive integers.In the usual notation, let A denote the class of the functions of the form
$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$
which are analytic in the open unit disc
$$D = \{z \in \mathbb{C} :| z |< 1\}.$$

Further, let $\mathcal{S}$ denote the subclass of all functions in A consisting of functions which are univalent in D (see details in [8], [22]).We know that every univalent function $f \in \mathcal{S}$ has an inverse f^{-1}, given by
$$f^{-1}(f(z)) = z \qquad (z \in U)$$
and
$$f\left(f^{-1}(w)\right) = w. \qquad \left(| w |< r_0(f) \,; r_0(f) \geq \tfrac{1}{4}\right)$$
The inverse function $f^{-1}(w) = g(w)$ is defined by
$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \tag{1.2}$$

The Koebe one quarter theorem ([8]) ensures that the image of $\mathcal{S}$ under every f from $\mathcal{S}$ contains a disc of radius $\frac{1}{4}$. If both of the functions f and f^{-1} are univalent in $\mathcal{S}$, then a function $f \in A$ is said to be bi-univalent in $\mathcal{S}$. We

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shall demonstrate by $\sum$ the class of bi-univalent functions in $\mathcal{S}$ given by the Taylor- Maclourin series expantion given by (1.1). The familiar Koebe function is not member of $\sum$. Many interesting examples of functions which are in the class $\sum$ (or not in $\sum$) can be found in the earlier works in Lewin [10] studied the class of bi_univalent functions obtaining the bound $|a_2| < 1.51$, Brannan and Clunie [5] conctectured that $|a_2| \leq \sqrt{2}$ and Netanyahu ([12]) proved that $\max |a_2| = \frac{4}{3}$, for $f \in \sum$. In recent years Srivastava et al. ([21]), Frasin and Aouf ([9]) investigated various subclasses of the bi-univalent function class $\sum$ and found estimates on the Taylor-Maclourin coefficient $|a_2|$ and $|a_3|$ for functions in these subclasses. But a lot of researcher proved some results within these coefficient for different classes (see [1–4, 6, 7, 11, 17, 19]). The problem of estimating coefficients $|a_n|$, for $n \geq 2$ is still an open problem. Recently Al-Shaqsi and Darus [20] defined a function $(G(n;z))^{-1}$ given by

$$G(n;z) * (G(n;z))^{-1} = \frac{z}{(1-z)^{\lambda+1}}, \qquad (\lambda > -1,\, n \in \mathbb{N})$$

and obtained the following linear operator

$$D_\lambda^n f(z) = (G(n,z))^{-1} * f(z). \tag{1.3}$$

As it is well known, $G(n;z)$ is the polylogarithm function given by

$$G(n;z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n}. \qquad (n \in \mathbb{C},\, z \in U) \tag{1.4}$$

For $n = -1$, $G(-1;z) = \dfrac{z}{1-z^2}$ is Koebe function. For more details about polylogarthms in theory of univalent functions, see Ponnusamy and Sabapath [15] and Ponnusamy [14]. By using the explicit form of the function $(G(n,z))^{-1}$, for $\lambda > -1$, we obtain

$$(G(n,z))^{-1} = \sum_{k=1}^{\infty} k^n \frac{(k+\lambda-1)!}{\lambda!\,(k-1)!}. \qquad (z \in U) \tag{1.5}$$

For $n, \lambda \in \mathbb{N}_0 = \{0,1,2,3,\ldots\}$ Al_Shaqsi and Darus [20] defined that

$$D_\lambda^n f(z) = z + \sum_{k=2}^{\infty} k^n \frac{(k+\lambda-1)!}{\lambda!\,(k-1)!} a_k z^k. \qquad (z \in U) \tag{1.6}$$

If we take $\lambda = 0$ in equation(1.6)then we obtain

$$D_0^n f(z) = D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k \tag{1.7}$$

which gives Sălăgean's differential operator [18]. For $n = 0$

$$D_\lambda^0 f(z) = D^\delta f(z) = z + \sum_{k=2}^{\infty} C(\delta,k) a_k z^k \tag{1.8}$$

where $C(\delta,k) = \binom{k+\delta-1}{\delta}$, $\delta \in \mathbb{N}_0$, which gives Ruscheweyh derivative operator [16]. It is obvius that the operator D_λ^n included two well known derivative operators. Also we have

$$D_0^1 f(z) = D_1^0 f(z) = zf^{'}(z). \tag{1.9}$$

Making use of the polylogarithm functionD_λ^n , we now introduce two new subclasses of Σ. We investigate estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses employing the techniques used earlier by Srivastava et al ([21]) and by Frasin and Aouf ([9]). Let P be the class of functions with positive real part consisting of all analytic functions $P : U \to \mathbb{C}$ satisfying $P(0) = 1$ and $R(P(z)) > 0$.

To prove our main result, we need the following lemma ([13]).

Lemma 1.1. *If $h \in P$ then $\mid c_k \mid \leq 2$ for each k, where P is the family of all functions h analytic in U for which $Re(h(z)) > 0, h(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \ldots$ for$z \in U$.*

2. Coefficient Bounds for the function class $\mathcal{H}_{\Sigma}^{n,\mu}(\lambda,\alpha)$

Definition 2.1. A function $f(z)$ given by (1.1) is said to be in the class $\mathcal{H}_{\Sigma}^{n,\mu}(\lambda,\alpha)$ if the following conditions are satisfied

$$f \in \Sigma, \left|arg\left(\frac{D_{\lambda}^{n}f(z)}{z}\right)^{\mu}\right| < \frac{\alpha\pi}{2} \quad (0<\alpha\leq 1, : \lambda, n, \mu \in \mathbb{N}_0, : z \in U) \tag{2.1}$$

and

$$\left|arg\left(\frac{D_{\lambda}^{n}g(w)}{w}\right)^{\mu}\right| < \frac{\alpha\pi}{2} \quad (0<\alpha\leq 1, : \lambda, n, \mu \in \mathbb{N}_0, : w \in U) \tag{2.2}$$

where the function g is given by the equality (1.2). In this study, we will find the estimates on the coefficients $|\ a_2\ |$ and $|\ a_3\ |$ for functions in the class $\mathcal{H}_{\Sigma}^{\mu}(\lambda,\alpha)$.

Remark 2.1. If we choose $n=0, \lambda=1$ and $\mu=1$ in Definition 2.1, then the class $\mathcal{H}_{\Sigma}^{n,\mu}(\lambda,\alpha)$ reduces to the class $\mathcal{H}_{\Sigma}^{\alpha}$ introduced and studied by Srivastava et al.[21] .

Theorem 2.1. *Let $f(z)$ given by (1.1) in the class $\mathcal{H}_{\Sigma}^{n,\mu}(\lambda,\alpha), 0<\alpha\leq 1$ and $\lambda, n, \mu \in \mathbb{N}_0$. Then*

$$|\ a_2\ | \leq \frac{2\alpha}{\sqrt{\alpha\mu 3^n(\lambda+1)(\lambda+2)+2^{2n}(\lambda+1)^2\mu(\mu-\alpha)}} \tag{2.3}$$

and

$$|\ a_3\ | \leq \frac{4\alpha}{3^n(\lambda+1)(\lambda+2)\mu} + \frac{4\alpha^2}{2^{2n}\mu^2(\lambda+1)^2}. \tag{2.4}$$

Proof. It follows from inequalities (2.1) and (2.2) that

$$\left(\frac{D_{\lambda}^{n}f(z)}{z}\right)^{\mu} = [P(z)]^{\alpha} \quad (z \in U) \tag{2.5}$$

and

$$\left(\frac{D_{\lambda}^{n}g(w)}{w}\right)^{\mu} = [Q(w)]^{\alpha} \quad (w \in U) \tag{2.6}$$

where $p(z)$ and $q(w)$ in P and have the forms

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \dots \tag{2.7}$$

and

$$q(z) = 1 + q_1 z + q_2 z^2 + q_3 z^3 + \dots \tag{2.8}$$

Now, equating the coefficients in equations (2.5) and (2.6), we find that

$$2^n\mu(\lambda+1)a_2 = \alpha p_1 \tag{2.9}$$

$$\frac{3^n(\lambda+1)(\lambda+2)}{2}\mu a_3 + \frac{\mu(\mu-1)}{2}2^{2n}(\lambda+1)^2 a_2^2 = \alpha p_2 + \frac{\alpha(\alpha-1)}{2}p_1^2 \tag{2.10}$$

$$-2^n\mu(\lambda+1)a_2 = \alpha q_1 \tag{2.11}$$

and

$$\frac{3^n(\lambda+1)(\lambda+2)}{2}\mu(2a_2^2-a_3) + \frac{\mu(\mu-1)}{2}2^{2n}(\lambda+1)^2 a_2^2 = \alpha q_2 + \frac{\alpha(\alpha-1)}{2}q_1^2. \tag{2.12}$$

From (2.9) and (2.11), we get

$$p_1 = -q_1 \tag{2.13}$$

and

$$2(2^{2n}\mu^2(\lambda+1)^2 a_2^2) = \alpha^2(p_1^2+q_1^2). \tag{2.14}$$

Also from (2.10),(2.12) and (2.14), we obtain

$$\begin{aligned} 3^n(\lambda+1)(\lambda+2)\mu a_2^2 + \mu(\mu-1)2^n(\lambda+1)^2 a_2^2 &= \alpha(p_2+q_2) + \frac{\alpha(\alpha-1)}{2}(p_1^2+q_1^2) \\ &= \alpha(p_2+q_2) + \frac{(\alpha-1)2^n\mu^2(\lambda+1)^2 a_2^2}{\alpha} \end{aligned}$$

Therefore, we have

$$a_2^2 = \frac{\alpha^2(p_2+q_2)}{3^n(\lambda+1)(\lambda+2)\alpha\mu + 2^{2n}(\lambda+1)^2\mu(\mu-\alpha)}$$

Applying Lemma 1.1. for the coefficients p_2 and q_2, we have

$$| a_2 | \leq \frac{2\alpha}{\sqrt{3^n(\lambda+1)(\lambda+2)\alpha\mu + 2^{2n}(\lambda+1)^2\mu(\mu-\alpha)}}$$

This gives the bound on $|a_2|$ as asserted in inequality (2.3). Next in order to find the bound on $|a_3|$, by subtracting (2.12) from (2.10), we find that

$$\begin{aligned} &3^n(\lambda+1)(\lambda+2)\mu a_3 - 3^n(\lambda+1)(\lambda+2)\mu a_2^2 \\ &= \alpha(p_2-q_2) + \frac{\alpha(\alpha-1)}{2}(p_1^2-q_1^2) \end{aligned} \tag{2.15}$$

It follows from (2.13), (2.14) and (2.15) that

$$3^n(\lambda+1)(\lambda+2)\mu a_3 - 3^n(\lambda+1)(\lambda+2)\mu \frac{\alpha^2(p_1^2+q_1)^2}{2^{2n+1}\mu^2(\lambda+1)^2} = \alpha(p_2-q_2)$$

thus,we have

$$3^n(\lambda+1)(\lambda+2)\mu a_3 = \alpha(p_2-q_2) + \frac{3^n(\lambda+1)(\lambda+2)\mu\alpha^2(p_1^2+q_1^2)}{2^{2n+1}\mu^2(\lambda+1)^2}$$

or equivalently,

$$a_3 = \frac{\alpha(p_2-q_2)}{3^n(\lambda+1)(\lambda+2)\mu} + \frac{\alpha^2(p_1^2+q_1^2)}{2^{2n+1}\mu^2(\lambda+1)^2}$$

Applying Lemma 1.1. for the coefficients p_1, p_2, q_1, q_2 we get

$$|a_3| \leq \frac{4\alpha}{3^n(\lambda+1)(\lambda+2)\mu} + \frac{4\alpha^2}{2^{2n}\mu^2(\lambda+1)^2}.$$

This completes the proof of Theorem (2.1). □

If we choose $n = 0, \lambda = 1$ and $\mu = 1$ in Theorem (2.1), then we reduce the result by Srivastava et al. [21], as follow:

Corollary 2.1. *Let the function function $f(z)$ given by (1.1) in the class $\mathcal{H}_\Sigma^\alpha(0 < \alpha \leq 1)$. Then*

$$|a_2| \leq \alpha\sqrt{\frac{2}{\alpha+2}}$$

and

$$|a_3| \leq \frac{\alpha(3\alpha+2)}{3}.$$

3. Coefficient Bounds for the Function Class $\mathcal{B}_\Sigma^{n,\mu}(\beta, \lambda)$

Definition 3.1. A function $f(z)$ given by (1.1) is said to be in the class $\mathcal{B}_\Sigma^{n,\mu}(\beta, \lambda)$ if the following conditions are satisfied

$$f \in \Sigma,\ Re\left(\frac{D_\lambda^n f(z)}{z}\right)^\mu > \beta \qquad (0 \leq \beta < 1,\ \lambda, n, \mu \in \mathbb{N}_0, z \in U) \tag{3.1}$$

and

$$Re\left(\frac{D_\lambda^n g(w)}{w}\right)^\mu > \beta \qquad (0 \leq \beta < 1,\ \lambda, n, \mu \in \mathbb{N}_0,\ w \in U) \tag{3.2}$$

where the function g is given by (1.2).

Remark 3.1. If we choose $n = 0, \lambda = 1$ and $\mu = 1$ in Definition (3.1), then the class $\mathcal{B}_{\Sigma}^{n,\mu}(\beta, \lambda)$ reduces to the class $\mathcal{H}_{\Sigma}(\beta), (0 \leq \beta < 1)$ introduced and studied by Srivastava et al. [21].

Theorem 3.1. *Let $f(z)$ given by (1.1) be in the class $\mathcal{B}_{\Sigma}(\beta, \lambda)$ $0 \leq \beta < 1$, and $\lambda, n, \mu \in \mathbb{N}_0$. Then*

$$| a_2 |\leq \sqrt{\frac{4(1-\beta)}{| 3^n(\lambda+1)(\lambda+2)\mu + \mu(\mu-1)2^n(\lambda+1)^2 |}} \tag{3.3}$$

and

$$|a_3| \leq \frac{4(1-\beta)}{3^n(\lambda+1)(\lambda+2)\mu + \mu(\mu-1)^2 2^n(\lambda+1)^2} + \frac{4(1-\beta)^2}{2^{2n}\mu^2(\lambda+1)^2} \tag{3.4}$$

Proof. It follows from 3.1 and 3.2 that there exist p and $q \in P$ such that

$$\left(\frac{D_\lambda^n f(z)}{z}\right)^\mu = \beta + (1-\beta)p(z) \qquad (z \in U) \tag{3.5}$$

and

$$\left(\frac{D_\lambda^n g(w)}{w}\right)^\mu = \beta + (1-\beta)q(w) \qquad (w \in U) \tag{3.6}$$

where $p(z)$ and $q(z)$ have the forms (2.7) and (2.8) respectively. By equating coefficients of the equations (3.5) and (3.6), we get

$$2^n(\lambda+1)\mu a_2 = (1-\beta)p_1 \tag{3.7}$$

$$\frac{3^n(\lambda+1)(\lambda+2)}{2}\mu a_3 + \frac{\mu(\mu-1)}{2}2^{2n}(\lambda+1)^2 a_2^2 = (1-\beta)p_2 \tag{3.8}$$

$$-2^n(\lambda+1)\mu a_2 = (1-\beta)p_1 \tag{3.9}$$

$$\frac{3^n(\lambda+1)(\lambda+2)}{2}\mu(2a_2^2 - a_3) + \frac{\mu(\mu-1)}{2}2^{2n}(\lambda+1)^2 a_2^2 = (1-\beta)p_2 \tag{3.10}$$

From (3.7) and (3.9), we have

$$p_1 = -q_1 \tag{3.11}$$

and

$$2.2^{2n}(\lambda+1)^2\mu^2 a_2^2 = (1-\beta)^2(p_1^2 + q_1)^2 \tag{3.12}$$

Also, from (3.8) and (3.10), we find that ,

$$3^n(\lambda+1)(\lambda+2)\mu a_2^2 + \mu(\mu-1)2^{2n}(\lambda+1)^2 a_2^2 = (1-\beta)(p_2+q_2)$$

Therefore, we have

$$a_2^2 = \frac{(1-\beta)(p_2+q_2)}{3^n(\lambda+1)(\lambda+2)\mu + \mu(\mu-1)2^{2n}(\lambda+1)^2} \tag{3.13}$$

and

$$|a_2^2| \leq \frac{(1-\beta)(|p_2|+|q_2|)}{3^n(\lambda+1)(\lambda+2)\mu + \mu(\mu-1)2^{2n}(\lambda+1)^2} \tag{3.14}$$

Applying Lemma1.1, we get desired result on the coefficient $|a_2|$ as asserted in (3.3). Next, in order to find the bound on $|a_3|$ by subtracting (3.10) from (3.8), we get

$$3^n(\lambda+1)(\lambda+2)\mu a_3 - 3^n(\lambda+1)(\lambda+2)\mu a_2^2 = (1-\beta)(p_2-q_2)$$

which, upon of the value of a_2^2 from (3.12), yields

$$3^n(\lambda+1)(\lambda+2)\mu a_3 = 3^n(\lambda+1)(\lambda+2)\mu\frac{(1-\beta)^2(p_1^2+q_1^2)}{2^{2n+1}(\lambda+1)^2\mu^2} + (1-\beta)(p_2-q_2).$$

Then,we have

$$a_3 = \frac{(1-\beta)(p_2+q_2)}{3^n(\lambda+1)(\lambda+2)\mu + \mu(\mu-1)2^{2n}(\lambda+1)^2} + \frac{(1-\beta)^2(p_1^2+q_1^2)}{2^{2n+1}(\lambda+1)^2\mu^2}$$

Applying Lemma 1.1 for the coefficients $p_1,\ q_1,\ p_2$ and q_2 we obtain

$$|a_3| \leq \frac{(1-\beta)\left(|p_2|+|q_2|\right)}{3^n(\lambda+1)(\lambda+2)\mu+\mu(\mu-1)2^{2n}(\lambda+1)^2}+\frac{(1-\beta)^2(|p_1|^2+|q_1|^2)}{2^{2n+1}(\lambda+1)^2\mu^2}$$

which is the desired estimate on the coefficient $|a_3|$ as asserted in (3.4). □

If we take $n=0,\ \mu=1 and \lambda=1$ in the Theorem (3.1), then we reduce $\mathcal{H}_\Sigma(\beta)(0\leq\beta<1)$ introduced and studied Srivastava et all , as follow:

Corollary 3.1. *([21]) Let the function function $f(z)$ given by (1.1) in the class $H_\Sigma(\beta)(0\leq\beta<1)$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{3}}$$

and

$$|a_3| \leq \frac{(1-\beta)(5-3\beta)}{3}.$$

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